Name: ______________________________

Student ID: ______________________________

Section: ______________________________

Instructor: ______________________________

Math 113 (Calculus 2)
Exam 2

9-13 October 2009

Instructions:

1. Work on scratch paper will not be graded.
2. Should you have need for more space than is allotted to answer a question, use the back of the page the problem is on and indicate this fact.
3. Simplify your answers. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, $\tan^{-1}(1)$, etc. must be simplified for full credit.
4. Calculators are not allowed.

For Instructor use only.

#	Possible	Earned		#	Possible	Earned
M.C.	36			12	8	
10 a-c	12			13	8	
10 d-f	12			14	8	
11	8			15	8	
Sub	68			Sub	32	
				Total	100	

Answers to MC: 1C 2C 3D 4A 5B 6B 7D 8C 9E

Multiple Choice (36 points). Fill in the answer to each problem on your scantron. Make sure your name, section and instructor is on your scantron.

1. $\displaystyle\int_0^1 te^{-t}\,dt =$

   A. 1 B. $1-\frac{1}{e}$ C. $1-\frac{2}{e}$ D. $1+\frac{1}{e}$ E. $1+\frac{2}{e}$

2. $\displaystyle\int_0^{\pi/2} \sin^3 x \cos^2 x\,dx =$

   A. 0 B. $\frac{1}{15}$ C. $\frac{2}{15}$ D. $\frac{1}{5}$ E. $\frac{4}{15}$

3. $\displaystyle\int_0^{\pi} \cos^2 x\,dx =$

   A. $\frac{\pi}{5}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$ E. π

4. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2+4}}$

   A. $-\frac{\sqrt{x^2+4}}{4x}+C$ B. $-\frac{\sqrt{x^2+4}}{x}+C$ C. $\frac{\sqrt{x^2+4}}{4x}+C$ D. $\frac{\sqrt{x^2+4}}{x}+C$

5. $\displaystyle\int_{-1}^0 \frac{dx}{x^2+2x+2} =$

   A. $\frac{\pi}{5}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$ E. π

6. $\displaystyle\int_1^2 \frac{dx}{(x+1)(x+2)} =$

   A. $\ln\frac{10}{9}$ B. $\ln\frac{9}{8}$ C. $\ln\frac{8}{7}$ D. $\ln\frac{7}{6}$ E. $\ln\frac{6}{5}$ F. $\ln\frac{5}{4}$ G. $\ln\frac{4}{3}$

7. $\displaystyle\int_0^\infty \frac{dx}{1+x^2} =$

A. $\frac{\pi}{5}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$ E. π

8. What is the integral definition of $\ln x$?

A. $\displaystyle\int_0^x \frac{1}{t}dt$ for $x > 0$

B. $\displaystyle\int_1^x \frac{1}{t}dt$ for $x > 1$

C. $\displaystyle\int_1^x \frac{1}{t}dt$ for $x > 0$

D. $\displaystyle\int_0^x \frac{1}{t}dt$ for all real numbers x

E. $\displaystyle\int_1^x \frac{1}{t^2}dt$ for $x > 0$

F. $\displaystyle\int_1^e \frac{1}{t}dt$ for $x > 0$

9. $\displaystyle\int \sec^3 x\, dx =$

A. $\frac{1}{2}\sec x \tan x + C$

B. $\frac{1}{2}\ln|\sec x + \tan x| + C$

C. $\frac{1}{2}(\sec x + \ln|\sec x|) + C$

D. $\frac{1}{2}(\csc x \cot x + \ln|\csc x - \cot x|) + C$

E. $\frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$

Short Answer. Fill in the blank with the appropriate answer. 4 points each. A correct answer gets full credit. You will need to show your work for partial credit.

10. (24 points)

(a) Use the integral definition of $\ln 2$ and the midpoint rule with $n = 2$ to approximate $\ln 2$.

$\frac{1}{2}(\frac{4}{5}+\frac{4}{7}) = \frac{24}{35}$ ____________

(b) If $f'(x) < 0$ and $f''(x) > 0$ for $a \leq x \leq b$, Order L_n, R_n, M_n and T_n where L_n is the left endpoint approximation, R_n is the right endpoint approximation, M_n is the midpoint rule, and T_n is the trapezoidal rule each using n subdivisions.

R_n < M_n < T_n < L_n

(c) If $\sin\theta = x$, find $\sin 2\theta$ in terms of x.

$2x\sqrt{1-x^2}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-x^2}$
$\sin 2\theta = 2\sin\theta\cos\theta = 2x\sqrt{1-x^2}$

(d) Evaluate $\displaystyle\int \frac{x^3+x+1}{x^2+1}dx$

$\dfrac{x^2}{2}+\tan^{-1}x+C$

$$\frac{x^3+x+1}{x^2+1} = x + \frac{1}{x^2+1}$$

(e) Circle the integrals that converge and put an X over the integrals that diverge.

A. $\displaystyle\int_1^\infty \frac{dx}{x^2}$ B. $\displaystyle\int_0^1 \frac{dx}{x^2}$ C. $\displaystyle\int_1^\infty \frac{3+\sin 2x}{x}dx$ D. $\displaystyle\int_0^1 \frac{3+\sin 2x}{\sqrt{x}}dx$

A and D converge. B and C diverge.

(f) A table for the function f is given. Use the table and Simpson's Rule with $n = 4$ to estimate $\displaystyle\int_0^2 f(x)\,dx$.

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	2.5	2.8	3.0	3.2	3.5

6

Show your work for problems 11-15. Each problem is worth 8 points.

11. Evaluate the integral $\int_{-1}^{3} \sqrt{3+2t-t^2}\, dt$.

$$\int_{-1}^{3} \sqrt{3+2t-t^2}\, dt = \int_{-1}^{3} \sqrt{3+1-(t^2-2t+1)}\, dt = \int_{-1}^{3} \sqrt{3+1-(t-1)^2}\, dt$$

Let $u = t-1, du = dt$. When $t = -1, u = -2$. When $t = 3, u = 2$.

$$\int_{-2}^{2} \sqrt{4-u^2}\, du = \text{Area of a half-circle of radius } 2 = 2\pi.$$

or make a trig substitution to get $4\int_{-\pi/2}^{\pi/2} \cos^2\theta\, d\theta = 2\pi$

12. Evaluate the integral $\int \sqrt{\frac{1+x}{1-x}} dx$

Multiplying numerator and denominator by $\sqrt{1+x}$, we have

$$\int \sqrt{\frac{1+x}{1-x}} dx = \int \frac{1+x}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} + \int \frac{x}{\sqrt{1-x^2}} = \sin^{-1} x - \sqrt{1-x^2} + C$$

13. Use the Comparison Theorem to determine whether the integral is convergent or divergent. $\int_0^\infty \frac{x^2}{x^5+7}\, dx$. Justify your reasoning.

$$\int_0^\infty \frac{x^2}{x^5+7}\, dx = \int_0^1 \frac{x^2}{x^5+7}\, dx + \int_1^\infty \frac{x^2}{x^5+7}\, dx.$$

The first integral is finite. For $1 \le x < \infty$, we have $\frac{x^2}{x^5+7} < \frac{x^2}{x^5} = \frac{1}{x^3}$. So the second integral converges by a comparison test with $\int_1^\infty \frac{1}{x^3}\, dx$. So the integral is convergent.

14. Evaluate the integral $\int \sin 8x \sin 5x \, dx$.

$$\sin 8x \sin 5x = \frac{1}{2}(\cos(8x - 5x) - \cos(8x + 5x)) = \frac{1}{2}(\cos 3x - \cos 13x)$$

$$\int \sin 8x \sin 5x \, dx = \frac{1}{2} \int (\cos 3x - \cos 13x) \, dx = \frac{1}{2}\left(\frac{\sin 3x}{3} - \frac{\sin 13x}{13}\right) + C =$$

$$\frac{\sin 3x}{6} - \frac{\sin 13x}{26} + C$$

15. Evaluate the integral $\int \frac{\sqrt{x^2 - 9}}{x^4} \, dx$.

Let $x = 3 \sec \theta$ and $dx = 3 \sec \theta \tan \theta \, d\theta$.

$$\int \frac{\sqrt{x^2 - 9}}{x^4} \, dx = \int \frac{\sqrt{9 \sec^2 \theta - 9}}{3^4 \sec^4 \theta} 3 \sec \theta \tan \theta \, d\theta = \frac{1}{9} \int \frac{\tan^2 \theta}{\sec^3 \theta} \, d\theta = \frac{1}{9} \int \sin^2 \theta \cos \theta \, d\theta =$$

$$\frac{1}{27} \sin^3 \theta + C = \frac{1}{27} \left(\frac{\sqrt{x^2 - 9}}{x}\right)^3 + C = \frac{(x^2 - 9)^{\frac{3}{2}}}{27x^3} + C$$